

# Detectability of Mode Resonances in Coalescing Neutron Star Binaries

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Inspirals of neutron star-neutron star binaries are a promising source of gravitational waves for gravitational wave detectors like LIGO. During the inspiral, the tidal gravitational field of one of the stars can resonantly excite internal modes of the other star, resulting in a phase shift in the gravitational wave signal. We compute using a Fisher-matrix analysis how large the phase shift must be in order to be detectable. For a  $1.4M_{\odot}, 1.4M_{\odot}$  binary the result is  $\sim 8.1, 2.9$  and  $1.8$  radians, for resonant frequencies of 16, 32 and 64 Hz. The measurement accuracies of the other binary parameters are degraded by inclusion of the mode resonance effect.

## I. INTRODUCTION AND SUMMARY

In neutron star - neutron star (NS-NS) binaries, each star exerts a tidal gravitational force on its companion as the stars inspiral. This force can resonantly drive internal modes of oscillation of the companion [2, 3, 4, 5, 6, 7, 8, 9, 10], which in turn can alter the emitted gravitational wave signal. The mode driving can be primarily dissipative, or adiabatic, or resonant; see for example the discussion in Ref. [9]. The modification to the emitted gravitational wave signal can potentially yield information about the internal structure of the stars, or impede the matched-filtering based detection of the events.

In this paper we will focus attention on resonant mode driving. To a good approximation, the effect of the mode resonance on the gravitational wave signal can be described by two parameters, the gravitational-wave frequency  $f_0$  (twice the orbital frequency) at which resonance occurs, and a constant phase shift parameter  $\Delta\Phi$ . If we denote by  $\Phi_0(f)$  the phase of the Fourier transform of the gravitational waveform neglecting mode driving, then the phase  $\Phi(f)$  including mode driving can be written as [9]

$$\Phi(f) = \begin{cases} \Phi_0(f) + (1 - f/f_0)\Delta\Phi & f_0 - f \gg \Delta f_{\text{res}} \\ \Phi_0(f) & f - f_0 \gg \Delta f_{\text{res}} \end{cases} \quad (1.1)$$

Here we have chosen the parameters of the unperturbed and perturbed waveforms so that the waveforms coincide after the resonance. Also  $\Delta f_{\text{res}}$  is the bandwidth of the resonance, which is sufficiently small<sup>1</sup> that we can neglect it for data-analysis purposes. Thus, the phase perturbation due to the resonance is a linear function of frequency which vanishes after the resonance, whose maximum value is  $\sim \Delta\Phi$ .

There are examples known of situations for which the

resonant phase shift  $\Delta\Phi$  is large compared to unity, which implies the effect should be easily detectable in the gravitational wave signal. However all such cases requires the spin frequency of the star to be larger than is thought to be likely for most NS-NS inspirals. Resonant excitation requires that the mode frequency be small compared with the natural frequency  $\omega \sim \sqrt{M/R^3}$  of the star, where  $M$  is the stellar mass and  $R$  the stellar radius. One class of modes with suitably small frequencies are  $g$ -modes; however the overlap integrals for these modes are so small that  $\Delta\Phi$  is small compared to unity [2, 3, 4]. Another class are the  $f$  and  $p$ -modes of rapidly rotating stars, where the inertial-frame frequency  $\omega_m$  can be much smaller than the corotating-frame frequency  $\sim \sqrt{M/R^3}$ . Ho and Lai [5] showed that the phase shifts due to these modes could be large compared to unity. However, the required NS spin frequencies are several hundred Hz, which is thought to be unlikely in inspiralling NS-NS binaries. A third class of modes are Rossby modes ( $r$ -modes) [11, 12], for which the restoring force is dominated by the Coriolis force. For these modes the mode frequency  $\omega_m$  is of order the spin frequency of the star, and thus can be suitably small [ $10 \text{ Hz} \lesssim \omega_m/(2\pi) \lesssim 100 \text{ Hz}$ ]. Ho and Lai [5] computed the Newtonian driving of these modes, and showed that the phase shift is small compared to unity. Ref. [9] showed that a larger phase shift is produced by post-Newtonian, gravitomagnetic driving, but that once again  $\Delta\Phi \gtrsim 1$  is only possible for spin frequencies of order several hundred Hz.

In this paper we compute how large the phase shift  $\Delta\Phi$  needs to be in order to be detectable. We confirm the prevailing expectation that the detectability criterion is  $\Delta\Phi \gtrsim 1$ . More specifically, we consider a simplified model of the gravitational wave signal [1] which depends only on 5 parameters: the masses  $M_1$  and  $M_2$  of the two stars, the time  $t_c$  of coalescence, the orbital phase  $\phi_c$  at coalescence, and an overall amplitude parameter  $\mathcal{A}$ <sup>2</sup>. Table I, from Ref. [1], shows the accuracy with which these parameters could be measured in the absence of

<sup>1</sup> From Eqs. (1.5) and (3.2) of Ref. [9], the bandwidth is

$$\Delta f_{\text{res}} \sim 0.1 \text{ Hz} \left( \frac{\mathcal{M}}{1.2 M_{\odot}} \right)^{5/6} \left( \frac{f_0}{10 \text{ Hz}} \right)^{11/6},$$

where  $\mathcal{M}$  is the chirp mass of the binary.

<sup>2</sup> For simplicity we neglect the influence of the spins of the NSs on the parameter extraction. Including spin will degrade somewhat the measurement accuracies.

TABLE I: The rms errors for signal parameters, neglecting spin effects and in the absence of resonances, assuming the advanced LIGO noise spectrum [1], and for a signal-to-noise ratio of 10. The units of the stellar masses  $M_1$  and  $M_2$  are solar masses, while  $\Delta t_c$  is in milliseconds.

| $M_1$ | $M_2$ | $\Delta t_c$ | $\Delta \phi_c$ | $\Delta M/M$ | $\Delta \mu/\mu$ |
|-------|-------|--------------|-----------------|--------------|------------------|
| 2.0   | 1.0   | 0.72         | 1.31            | 0.004%       | 0.39%            |
| 1.4   | 1.4   | 0.71         | 1.28            | 0.004%       | 0.41%            |
| 10    | 1.4   | 1.01         | 1.63            | 0.020%       | 0.54%            |
| 15    | 5.0   | 1.44         | 2.02            | 0.113%       | 1.50%            |
| 10    | 10    | 1.43         | 1.98            | 0.160%       | 1.90%            |

TABLE II: The rms errors for signal parameters, as in Table I, but including the effects of resonances. The resonant frequency  $f_0$  is in Hz. The quantity  $\Delta(\Delta\Phi)$  is the rms measurement error in the phase shift parameter  $\Delta\Phi$ .

| $M_1$ | $M_2$ | $f_0$ | $\Delta t_c$ | $\Delta \phi_c$ | $\Delta M/M$ | $\Delta \mu/\mu$ | $\Delta f_0$ | $\Delta(\Delta\Phi)$ |
|-------|-------|-------|--------------|-----------------|--------------|------------------|--------------|----------------------|
| 2.0   | 1.0   | 16    | 0.81         | 1.68            | 0.007%       | 0.56%            | 24           | 8.1                  |
| 1.4   | 1.4   | 16    | 0.80         | 1.65            | 0.008%       | 0.58%            | 24           | 8.1                  |
| 10    | 1.4   | 16    | 1.20         | 2.22            | 0.040%       | 0.82%            | 24           | 8.6                  |
| 15    | 5.0   | 16    | 1.82         | 2.89            | 0.232%       | 2.44%            | 24           | 9.2                  |
| 10    | 10    | 16    | 1.81         | 2.85            | 0.318%       | 3.14%            | 24           | 9.2                  |
| 2.0   | 1.0   | 32    | 0.76         | 1.46            | 0.007%       | 0.47%            | 17           | 2.9                  |
| 1.4   | 1.4   | 32    | 0.75         | 1.43            | 0.007%       | 0.48%            | 17           | 2.9                  |
| 10    | 1.4   | 32    | 1.10         | 1.86            | 0.033%       | 0.64%            | 18           | 2.9                  |
| 15    | 5.0   | 32    | 1.64         | 2.36            | 0.165%       | 1.82%            | 18           | 2.9                  |
| 10    | 10    | 32    | 1.61         | 2.32            | 0.224%       | 2.33%            | 18           | 2.9                  |
| 2.0   | 1.0   | 64    | 0.97         | 2.44            | 0.005%       | 0.85%            | 23           | 1.8                  |
| 1.4   | 1.4   | 64    | 0.95         | 2.39            | 0.005%       | 0.88%            | 23           | 1.8                  |
| 10    | 1.4   | 64    | 1.59         | 3.47            | 0.030%       | 1.31%            | 23           | 2.1                  |
| 15    | 5.0   | 64    | 2.48         | 4.55            | 0.202%       | 3.87%            | 23           | 2.5                  |
| 10    | 10    | 64    | 2.45         | 4.45            | 0.280%       | 4.93%            | 23           | 2.5                  |

mode excitation, for an event with signal-to-noise ratio of 10, and assuming the advanced LIGO noise spectrum.

Next, we enlarge the signal parameter space to include the two parameters  $f_0$ , the gravitational wave frequency at resonance, and  $\Delta\Phi$ , the phase shift parameter. We compute measurement accuracies for binaries with masses  $(2M_\odot, 1M_\odot)$ ,  $(1.4M_\odot, 1.4M_\odot)$ ,  $(10M_\odot, 1.4M_\odot)$ ,  $(15M_\odot, 5M_\odot)$ ,  $(10M_\odot, 10M_\odot)$  and resonant frequencies  $f_0 = 16$  Hz, 32 Hz and 64 Hz with  $\Delta\Phi=1$ . The results are given in Table II. The entries in the last column of this table are the minimum values of  $\Delta\Phi$  necessary for detectability of the resonance effect (see Sec. II). We see that for a  $1.4M_\odot, 1.4M_\odot$  binary the minimum detectable values of  $\Delta\Phi$  are  $\sim 8.1, 2.9$  and  $1.8$  radians, for resonant frequencies of 16, 32 and 64 Hz.

We conclude that it is unlikely that mode resonances will be detectable by LIGO, unless the spins of the neutron stars are anomalously large. Our results also show that resonances at higher frequencies are easier to detect.

## II. DETAILS OF ANALYSIS

We start by reviewing the Fisher matrix formalism for computing parameter measurement accuracies; see for example Ref. [1]. The inner product used on the vector space of signals  $h(t)$  is given by:

$$(h_1 | h_2) = 2 \int_0^\infty \frac{\tilde{h}_1^*(f)\tilde{h}_2(f) + \tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df, \quad (2.1)$$

where  $\tilde{h}_1$  and  $\tilde{h}_2$  are the Fourier transform of two signals  $h_1$  and  $h_2$  respectively, and  $S_n(f)$  is the one-sided noise spectral density. We assume the model of the advanced LIGO noise curve used in [1]

$$S_n(f) = \begin{cases} \infty & f < 10 \text{ Hz}, \\ S_0 [(f_0/f)^4 + 2(1 + (f^2/f_0^2))] & f > 10 \text{ Hz} \end{cases} \quad (2.2)$$

where  $S_0$  is a constant whose value is unimportant for this paper, and  $f_0 = 70$  Hz. Given a signal model  $h = h(t, \theta^i)$ , where  $\theta^i$  are the parameters of the signal, the Fisher information matrix is defined by

$$\Gamma_{ij} \equiv \left( \frac{\partial h}{\partial \theta^i} \middle| \frac{\partial h}{\partial \theta^j} \right), \quad (2.3)$$

and the rms measurement error in the parameter  $\theta^i$  is

$$\sqrt{\langle (\Delta\theta^i)^2 \rangle} = \sqrt{\Sigma^{ii}}, \quad (2.4)$$

where  $\Sigma \equiv \Gamma^{-1}$ .

We next discuss our assumed form of the gravitational wave signal. In the absence of resonances, we use the model of Ref. [1]:

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\Phi_0(f)}, \quad (2.5)$$

where

$$\begin{aligned} \Phi_0(f) = & 2\pi f t_c - \phi_c - \pi/4 + \frac{3}{4}(8\pi\mathcal{M}f)^{-5/3} \\ & \times \left[ 1 + \frac{20}{9} \left( \frac{743}{336} + \frac{11\mu}{4M} \right) x - 16\pi x^{3/2} \right] \end{aligned} \quad (2.6)$$

Here  $M$  is the total mass of the binary,  $\mu$  is the reduced mass,  $\mathcal{M}$  is the chirp mass, and  $x = (\pi M f)^{1/3}$ . We assume the signal shuts off at a frequency of  $f = (6^{3/2}\pi M)^{-1}$ . To include the effect of resonances, we replace the phase  $\Phi_0(f)$  in Eq. (2.5) with the phase  $\Phi(f)$  given by Eq. (1.1), which depends in addition on the two parameters  $f_0$  and  $\Delta\Phi$ . [We approximate the bandwidth  $\Delta f_{\text{res}}$  of the resonance to be zero.] Using this signal model given by the seven parameters  $\mathcal{A}$ ,  $t_c$ ,  $\phi_c$ ,  $\mu$ ,  $M$ ,  $f_0$  and  $\Delta\Phi$ , we numerically compute the Fisher matrix (2.3), and invert the matrix to obtain the parameter measurement accuracies shown in Table II.

In these computations, we use the value  $\Delta\Phi = 1$ . The phase perturbation will be detectable when  $\Delta\Phi \gtrsim \Delta(\Delta\Phi)$ . From the form of the gravitational wave signal

it follows that the rms error  $\Delta(\Delta\Phi)$  is independent of the value of  $\Delta\Phi$ . Therefore the minimum value of  $\Delta\Phi$

necessary for detection is given by the computed value of  $\Delta(\Delta\Phi)$ , i.e. the last column in Table II.

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- [1] C. Cutler, E. E. Flanagan, Phys. Rev. D 49, 2658 (1994).
  - [2] A. Reisenegger and P. Goldreich, Astrophys. J. 426, 688 (1994).
  - [3] D. Lai, Mon. Not. Roy. Astr. Soc. 270, 611 (1994).
  - [4] K. D. Kokkotas and G. Schafer, Mon. Not. Roy. Astr. Soc. 275, 301 (1995)
  - [5] W. C. G. Ho and D. Lai, Mon. Not. Roy. Astr. Soc. 308, 153 (1999).
  - [6] L. Gualtieri et al., Phys. Rev. D 64, 104007 (2001).
  - [7] J. A. Pons et al., Phys. Rev. D 65, 104021 (2002).
  - [8] E. Berti et al., Phys. Rev. D 66, 064013 (2002).
  - [9] E. Racine, E. E. Flanagan, gr-qc/0601029.
  - [10] D. Lai, Y. Wu, Phys. Rev. D **74**, 024007 (2006).
  - [11] J. Provost, G. Berthomieu, and A. Rocca, Astron. and Astrophys. 94, 126+ (1981).
  - [12] H. Saio, Astrophys. J. 256, 717 (1982).